



Independent Study

Development of simulation module in Microsoft Project

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ABSTRACT

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Simulation has been applied recently in project scheduling. It provides possible outcomes from various uncertainty factors causing a project delay and helping project practitioner to avoid the risk of it.

Simulation must be performed in a computer program due to its calculation complexity and specific procedure, e.g., random number generation. Unfortunately, the popular computer program for project management, Microsoft Project, does not include the simulation function. To apply the simulation into the project, Microsoft Project user may perform a simulation on another program such as Microsoft Excel, by using two program to generate the result, consequently, unnecessary tedious task of copying the data from one to another program occurred. Thus, developing simulation software in Microsoft Project is helping and avoiding such situation.

The basic structure of the program needs 3 parts: input, processing and output. In the first part, Microsoft Project is the platform for receiving the data from user, probability distribution and task duration. Second part is the code, random number generation which is the main program of simulation, by using inverse transform. Finally, the output part will be represented on Microsoft Excel as statistical data, e.g. mean, confidence interval and possibility of being critical of each task.

As result from running the simulation program, simulation is more flexible in inputting the data, more choices of probability comparing to PERT analysis approach (only Beta distribution is applied), and more possible outcome of critical task occurrence while PERT providing only one critical path.

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Independent Study Advisor's signature

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INTRODUCTION

1.1 Background

A primary target of project management procedure is to manage a well-planned project duration. The duration of each task or activity in project must be properly settled and managed. The most popular tools which are used nowadays in project management are CPM (Critical Path Method) and PERT (Program Evaluation and Review Technique).

CPM (Critical Path Method)

is the tool used to calculate project duration by activities times in the project assumed to be known (deterministic). The project duration is then calculated from network analysis technique. The result from CPM is total duration of the project and critical path which both results are a guideline for planning, managing, and control the project [1] [2].

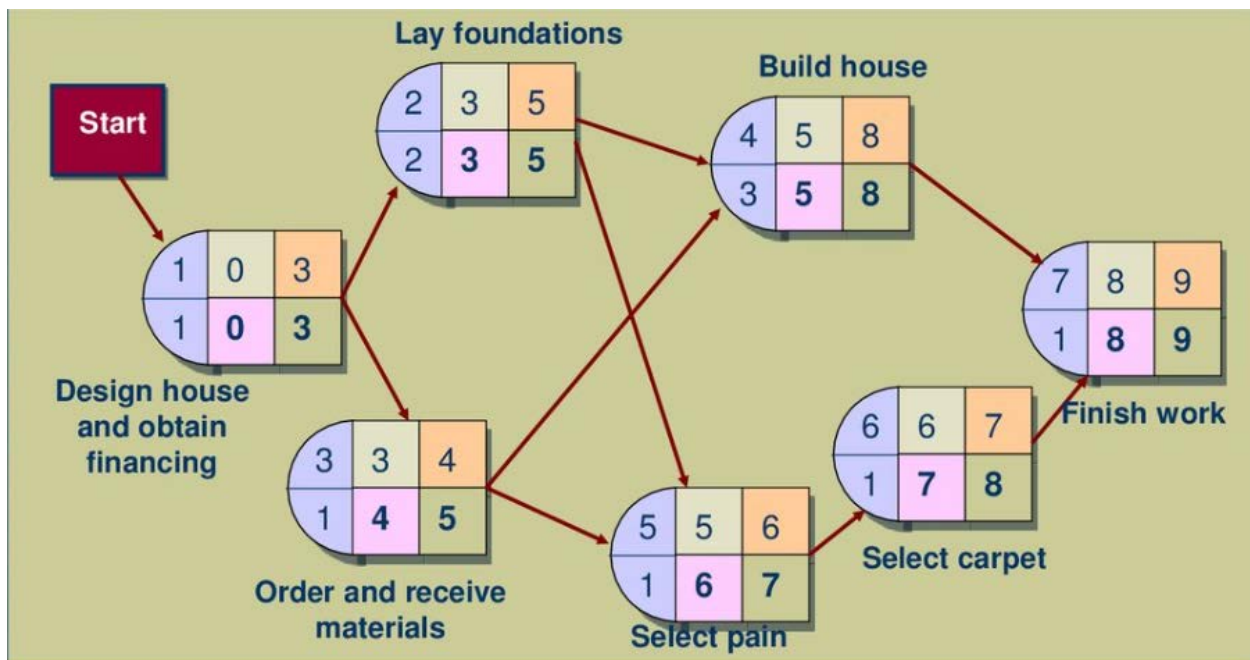


Figure 1.1 : Example of CPM, AON Network construction[John Wiley & Sons,Inc 2006]

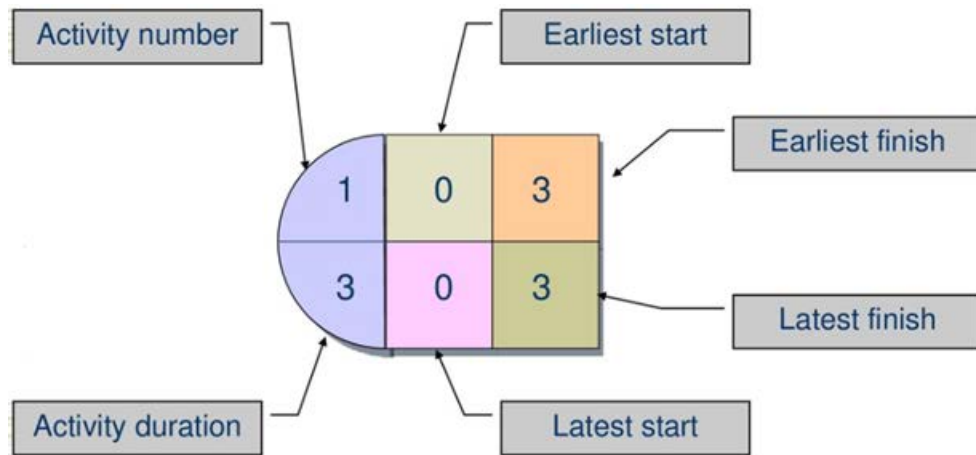


Figure 1.2 : Node configuration [John Wiley & Sons,Inc 2006]

PERT (Program Evaluation and Review Technique)

is the tool used to calculate project duration by activities time in the project estimated from the average and variance. The project duration is the sum of each estimated activity duration. The longest duration is the critical path of the project[1] [2].

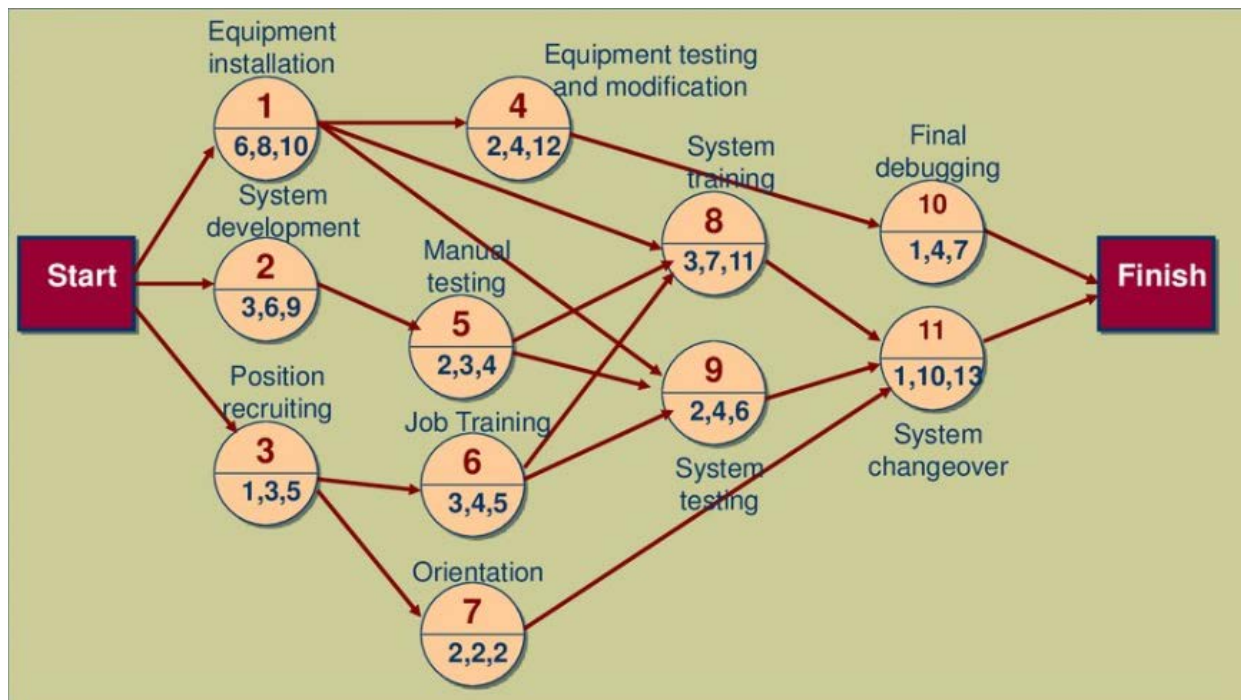


Figure 1.3 : Example of project network with probabilistic time estimation [Adopt from : John Wiley & Sons,Inc 2006]

ACTIVITY	TIME ESTIMATES (WKS)			MEAN TIME	VARIANCE
	<i>a</i>	<i>m</i>	<i>b</i>	<i>t</i>	σ^2
1	6	8	10	8	0.44
2	3	6	9	6	1.00
3	1	3	5	3	0.44
4	2	4	12	5	2.78
5	2	3	4	3	0.11
6	3	4	5	4	0.11
7	2	2	2	2	0.00
8	3	7	11	7	1.78
9	2	4	6	4	0.44
10	1	4	7	4	1.00
11	1	10	13	9	4.00

Figure 1.4 : PERT Activity time estimation [John Wiley & Sons, Inc 2006]

Nowadays computer program is the importance tools to help us calculating the project duration quickly and accurately especially in a large complex project. The project management software which is widely used today is Microsoft Project.



Figure 1.5 : Top 20 project management software [<https://www.capterra.com>]

The basic required information of using Microsoft Project are task name and its duration, task predecessor and start-finish time. Software will generate the project duration in task sheet and Gantt Chart according to previous information. The critical path will be automatically calculated and shown in this Gantt Chart

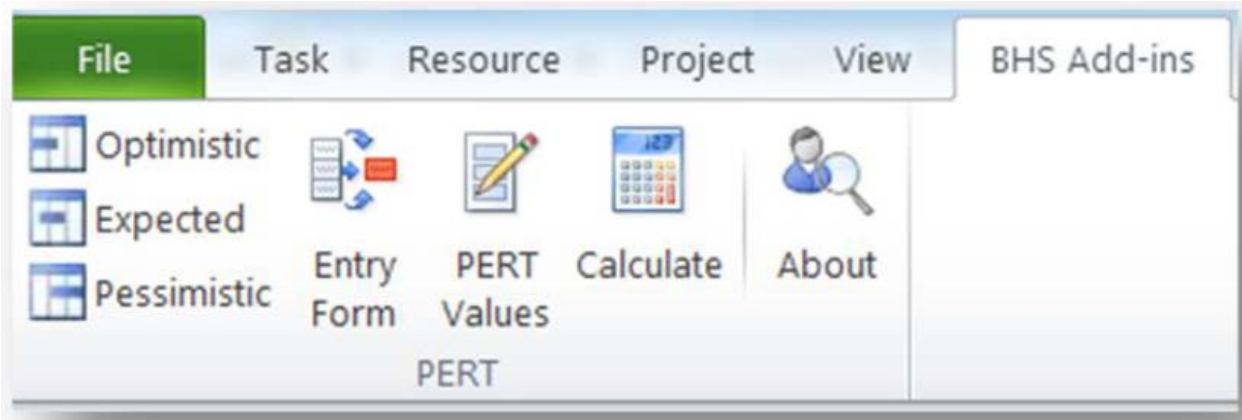


Figure 1.6 : PERT add-in menu ribbon in Microsoft Project program

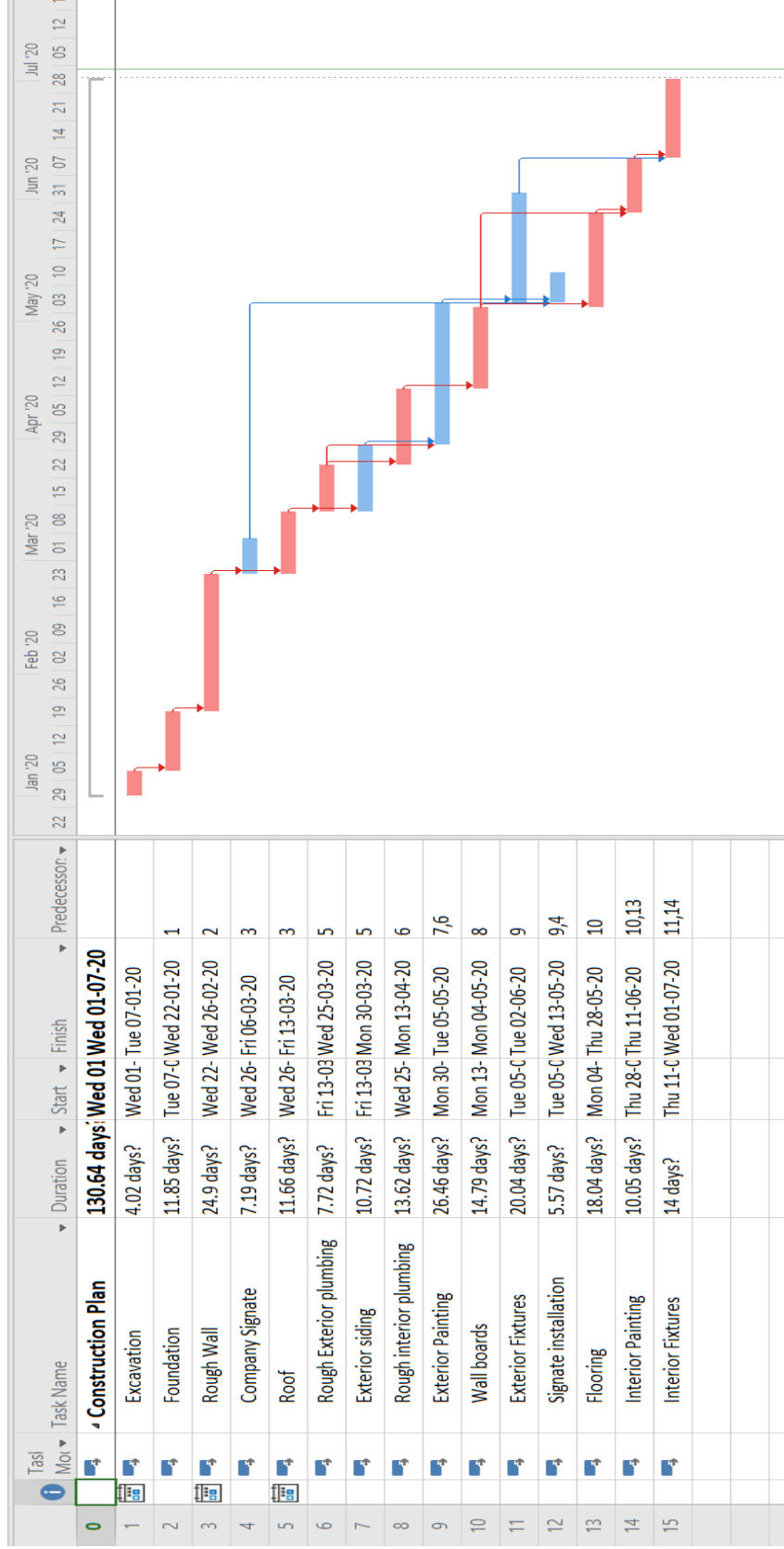
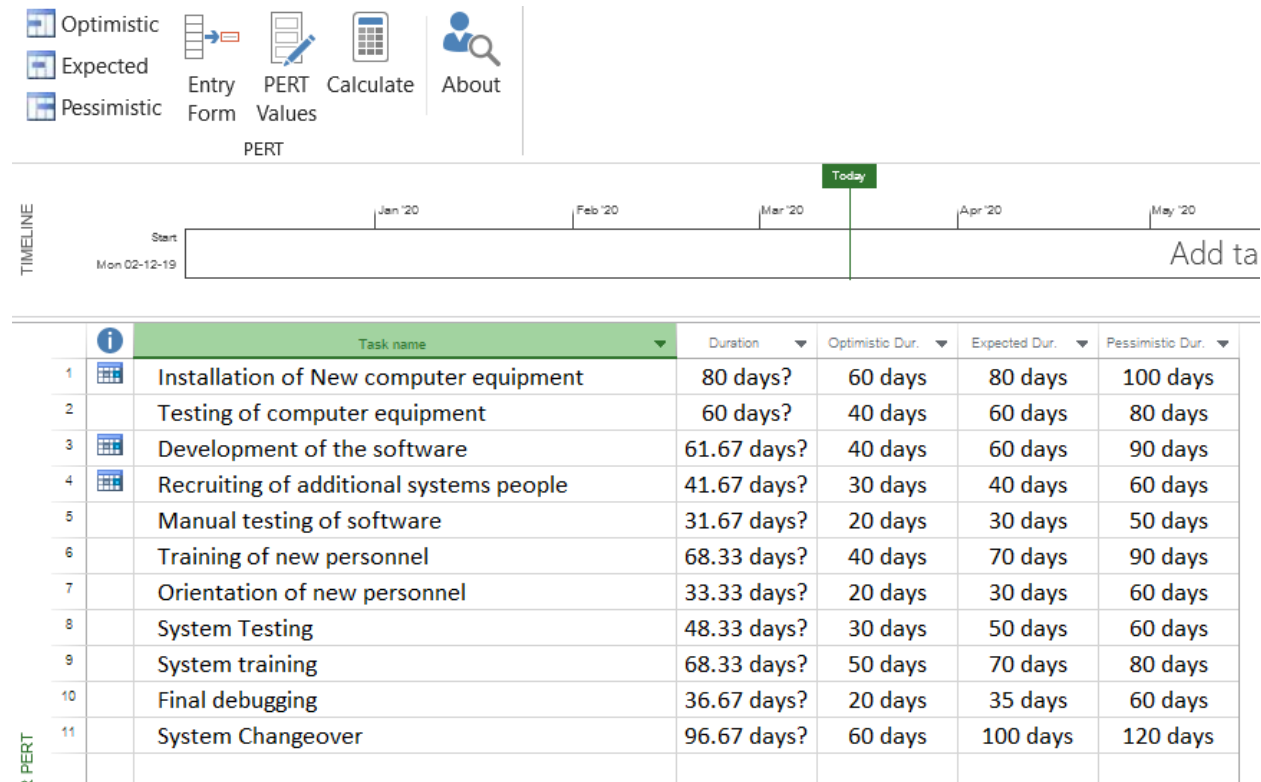


Figure 1.7 : Gantt chart view in

Microsoft Project also has a free add-in package of PERT. The task sheet of PERT mode is required Optimistic, Expected and Pessimistic duration of each task to generate the result.



	Optimistic	Expected	Pessimistic
Entry Form	PERT Values	Calculate	About

Task name	Duration	Optimistic Dur.	Expected Dur.	Pessimistic Dur.
1 Installation of New computer equipment	80 days?	60 days	80 days	100 days
2 Testing of computer equipment	60 days?	40 days	60 days	80 days
3 Development of the software	61.67 days?	40 days	60 days	90 days
4 Recruiting of additional systems people	41.67 days?	30 days	40 days	60 days
5 Manual testing of software	31.67 days?	20 days	30 days	50 days
6 Training of new personnel	68.33 days?	40 days	70 days	90 days
7 Orientation of new personnel	33.33 days?	20 days	30 days	60 days
8 System Testing	48.33 days?	30 days	50 days	60 days
9 System training	68.33 days?	50 days	70 days	80 days
10 Final debugging	36.67 days?	20 days	35 days	60 days
11 System Changeover	96.67 days?	60 days	100 days	120 days

Figure 1.8 : PERT analysis task table for user to fill duration in Microsoft Project program

1.2 Problem Statement

From the preceding discussion, CPM and PERT yield the same result which is the major critical path of the project. All activities that fall on critical path will be closely monitor and carefully managed to be started and finished on schedule, since the major effect of project lateness depending on the activities that fall on this path.

The main problem of using these tools are specific critical path result; only major critical path is shown from calculation. Other activities that are not shown in critical path, indeed, has some probability of being critical due to its variation or uncertainty factor. Sometimes such activities are the main cause of project duration delay[1]. Thus, the project manager must aware of the other unspecified critical activities by monitoring the probability of being critical of it. To find the probability of being critical, the tool called “Project Simulation” will be applied.

However, the latest version of Microsoft Project software doesn’t have a function to simulate the time and finding the probability of being critical of each activity and to estimate the project duration based on those critical activities.

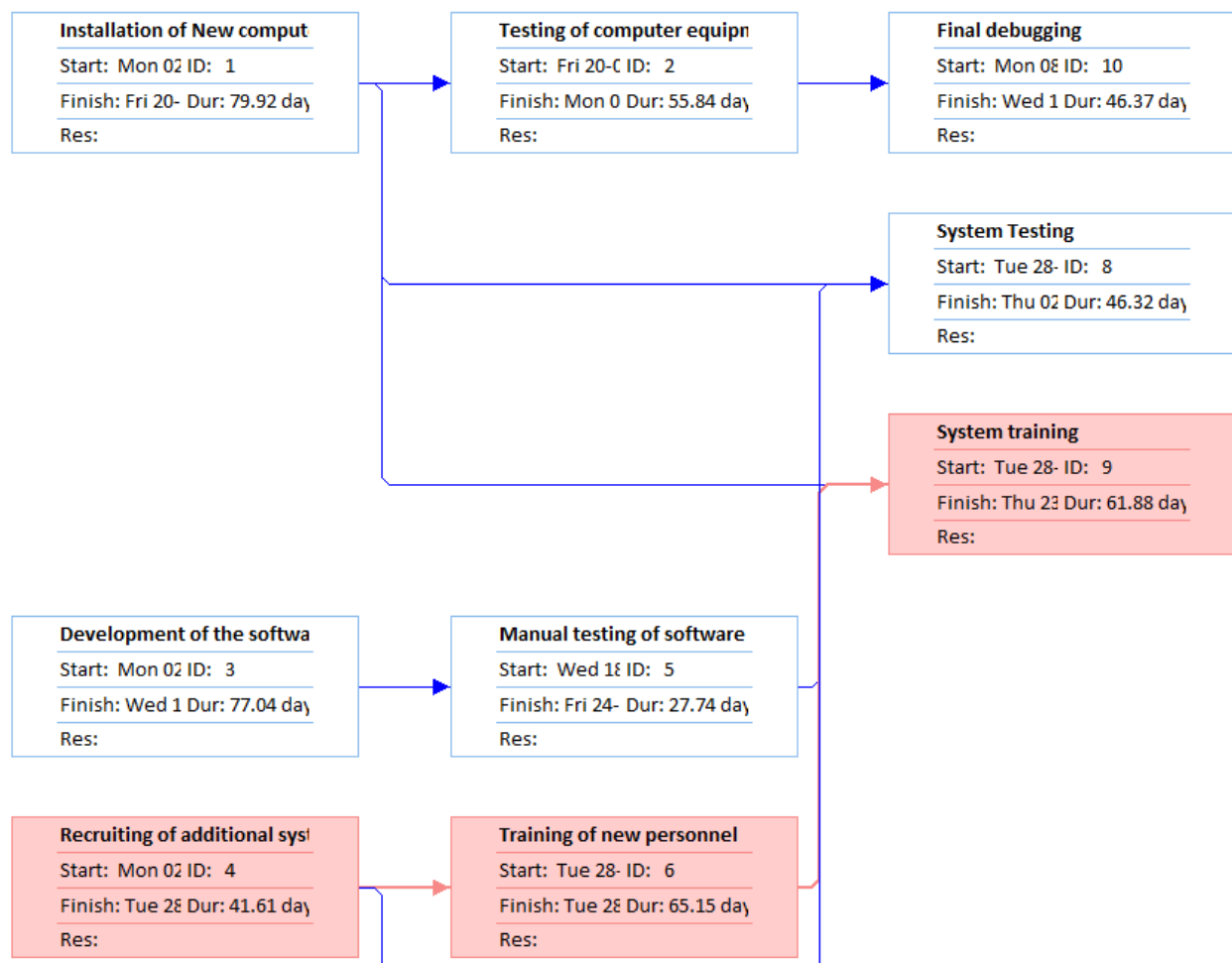


Figure 1.9 : Network diagram window

Therefore, the development of simulation software is an interesting option to improve the program to solve this problem.

1.3 Objective

The objective of this research is to develop the simulation program with Microsoft Project software environment. This development software must be able to simulate the time of each activity and give us the total duration, and able to evaluate the probability of being critical of every activity. This software will help the user enhance more results than standard version of Microsoft Project.

1.4 Conceptual Framework

The conceptual framework of this research is the function of generating random number regarding the probability distribution of each activity duration which all are input information from the current platform of Microsoft Project.

The conceptual framework detail is described in 3 parts as the program module.

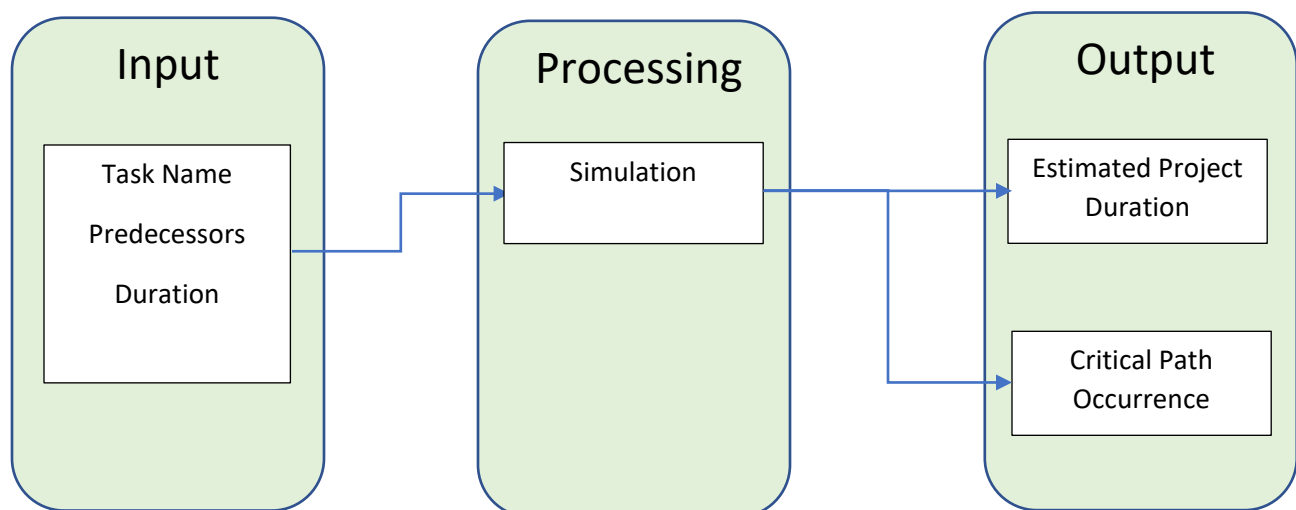


Figure 1.10 : Conceptual Framework of simulation program

1.4.1 Input

Microsoft Project (Standard)

- Task Name
- Predecessors
- Task duration
- Start-Finish date (optional)

New Software (Add-in)

- Probability distribution

Random number generation will be calculated based on probability distribution

- Min,Average,Max duration

The important parameters to calculate the random number

- Sample Size

Replication of simulation is needed to ensure that all possibility random values are considered

- Confidence Interval (%)

The estimated duration is calculated from the sample means and yield the result as interval form based on the input confidence interval(%). This help the user to claim that how many percent the result is correct and rely on.

1.4.2 Processing

is the main part of the new software. The software will create the random number between 0 and 1 (0,1) and send it to inverse function, to generate the random duration. Next, the software will send the result back to total duration function which complied by Microsoft Project program.

1.4.3 Output

is also the main part of the new software. Microsoft Excel program is chosen for the main part of presenting the result. After software complete the processing part, the average duration value will be calculated and shown in Microsoft Excel in the numeric form and chart, also the critical task occurrence result.

Microsoft Excel

- The estimated project duration of each repetition.
- The estimated project duration in confidence interval form
- Chart of estimated project duration versus percentage.
- Chart of estimated project duration in confidence interval (shaded area)
- Critical task occurrence.

Microsoft Project

- Average and variance duration of each task (optional)

1.5 Delimitation

The delimitation of this research is described in 3 parts as the program module.

1.5.1 Input

- This new software does not check the probability distribution type is fit for input data or not, only checking the correctness of data, e.g. the duration should be greater than zero
- Recurrence task cannot be computed in this new software.
- Project calendar, workdays, holidays, and others basic function in Microsoft Project cannot be set from this new software. If user need to set a calendar, they must use the Microsoft Project menu.

- Only probability distribution as detailed below are used in this software

- Normal Distribution
- Beta Distribution
- Triangular Distribution
- Uniform Distribution
- Exponential Distribution
- Weibull Distribution

including constant duration for which activity that its time is known

- Constant

1.5.2 Processing

- No seed function available for user to set the start random value for simulation since the purpose of running simulation is not for testing the model.

- One random variable will be used for entire project in each iteration of simulation, i.e. the random variable of each iteration is the same for all activities before being executed in inverse function of each distribution.

- The unit of time is set to be “days”.

- Resource, manhours and budget that are filled in Microsoft Project are not considered in this simulation.

1.5.3 Result

- Each estimated duration value is an average from sampling that was ran and collect in each iteration of simulation

- The critical task occurrence result is just only the value of possibility of being critical for all loop of simulation, not relating to any random number.

- Estimated duration and critical task occurrence result are not related

1.6 Scope

- 1.6.1 This software will be developed on Microsoft Project and Microsoft Excel version 2016
- 1.6.2 The designed of UI (User Interface) of this software is created from standard tools of VBA Microsoft Office only.
- 1.6.3 Some function and command in Microsoft Project will be used in this software such as Custom Field Duration, Custom Field Text. Other data in these fields will be overwritten every time when running this software and cannot be edited by manual.
- 1.6.4 The summary duration which is shown in task view will be updated every time when running this software; the existing data will be lost. Creating new file for running this software is recommended to avoid losing the existing data.
- 1.6.5 User has to set project calendar in Microsoft Project before running the software.

LITERATURE REVIEW

2.1 PERT and CPM

PERT and CPM are widely used in project time management. These tools help project managers generating overall project duration based on critical task, so they can decide and manage the project properly [8]. The principle of CPM and PERT are summarized as follow

2.1.1 CPM (Critical Path Method)

CPM was developed by Dupont, Inc and Sperry-Rand in 1950s, to manage industry project. A few years later, It was also widely used in construction project[2].

The procedure of using CPM are

1. Creating project network by using, for example, AON network
2. Finding earliest starting and finishing time by using forward pass method
3. Finding latest starting and finishing time by using backward pass
4. Finding project critical path, the longest duration path (No slack activity)

CPM: Calculating the slack

Slack: how much “wobble room” you have for an activity

$$\text{Slack} = \text{LS} - \text{ES} = \text{LF} - \text{EF}$$

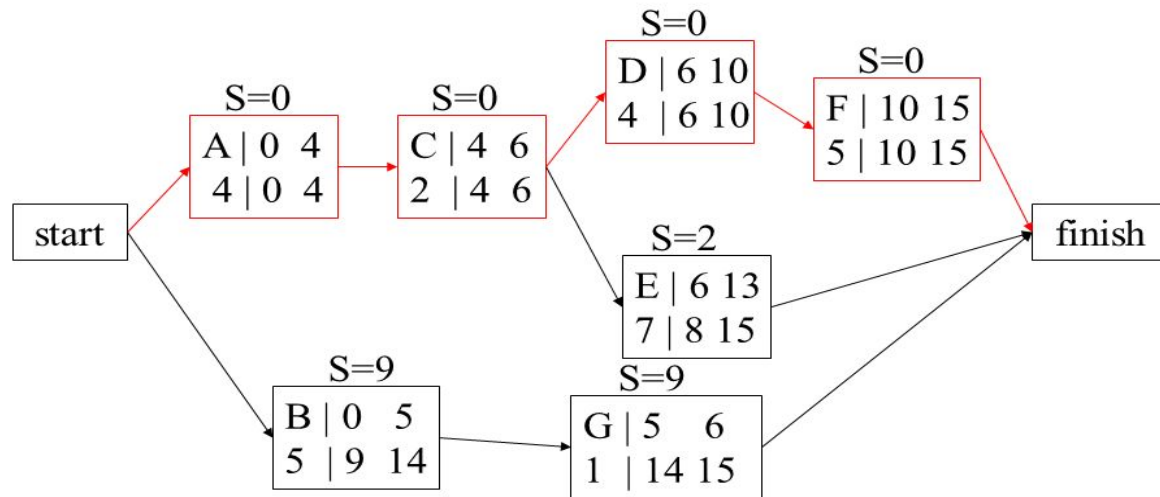


Figure 2.1 : Example of CPM method

Main purpose of using CPM

1. To find the project duration,
2. To settle each activity the proper starting and finishing time (Earliest and latest time of starting and finishing time).
3. To find a correlation between activity and duration.

Peculiarity and limitation of CPM

1. All activities duration must be known (Deterministic), mostly from experience or expert of that field.
2. Assume all resources is unlimited (No variation or uncertainty occurs)
3. Only one critical path given from calculation.

2.1.2 PERT (Program Evaluation and Review Technique)

PERT was developed by U.S. Navy, BoozAllen Hamilton and Lockheed Corporation and used in Polaris missile program, since the duration of some activities can be difficult to make especially for the new project [1][2]. PERT calculation emphasizing on the value of tree point estimation of each activity.

The procedure of using PERT are

1. Identify Most likely(m_i), Pessimistic(b_i), Optimistic(a_i) of each activity in project [2].
2. Calculate means and variance by substituting all values in following equation [2]

Expected duration

$$t_i = \frac{a_i + 4m_i + b_i}{6}$$

Variance

$$v_i = \frac{(b_i - a_i)^2}{36}$$

3. Find the total duration by summing all activities duration. The longest duration is the critical path [2]

Peculiarity and limitation of PERT

1. Most likely, Pessimistic and Optimistic Duration are needed for PERT calculation [2].
2. Critical path is the longest average duration. It will be used as a deadline of project duration which is not included possibility of being critical of another activities [2].
3. Normally PERT are performed based on Normal and Beta distribution[2]

Example of PERT is shown in Figure 1.3 and 1.4

Nowadays the project management computer program package as Microsoft Project include PERT function (see Figure 1.7 and 1.8)

Today, the widely used tools for project management is Monte Carlo Simulation, to find any possible risk of project lateness and reduce it possibility broader than PERT method which is performed under Beta distribution [8].

2.2 Simulation

Learning behavior of simple system may be used the Logical relationships or Mathematical approach which called Analytical Solution. However, in real-world problem, the system is more complex and cannot be applied approaches mentioned earlier. The simulation approach is another best choice for such system [3]

Simulation is the technique of system performance measurement when one or more independent variables are uncertain. If the independent variables are random variable then dependent variables are also random variable. Therefore the purpose of simulation is describing the behavior of independent variable from all possibility of independent variable[1].

Today, Monte Carlo Simulation is widely used to analyze the risk of project especially in project duration by generating random number between 0 and 1 (0,1) following probability distribution of each activity duration of each simulation process. After finishing all simulation process, average and variance of all activities will be calculated and summed for total project duration. [10].

2.3 Probability Distribution

The probability distribution and statistic play an importance role in studying and designing the simulation; the behavior of random input variable follows a specific distribution corresponding to specific module in system such as the processing time, the waiting time in queue and the service time etc [1][5].

Random Variable

Random variable is interesting value or characteristic that uncertainty due to some variations occurred from uncontrollable system or random experiment. The set of all possible outcomes of experiment is called the sample space[6]

Probability distribution is divided into 2 groups according to the type of random variable.

1. Discrete Probability distribution

is defined by the relationship between a random number and probability which is countable number[6].

2. Continuous Probability distribution

is defined by the relationship between a random number and probability which is real number[6].

In this study, the continuous probability distribution will be used.

Probability Density Function (PDF), Cumulative Distribution Function(CDF), Mean and Variance of continuous probability distribution in this study are shown in Appendix.

2.4 Random Numbers Generation

Algorithm is required for computer simulation to generate random numbers called pseudorandom number[4].

Pseudorandom Number

A sequence of pseudorandom numbers, U_i , is a deterministic sequence of number in (0,1) having the same relevant statistical properties as a sequence of truly random $U(0,1)$ numbers[4].

The algorithm for generating random number in this study used the random number generation algorithm of YASAI (Yet Another Simulation Added-in) program in Microsoft Excel, by creating a new value of each iteration from the value that is created in the earlier iteration.

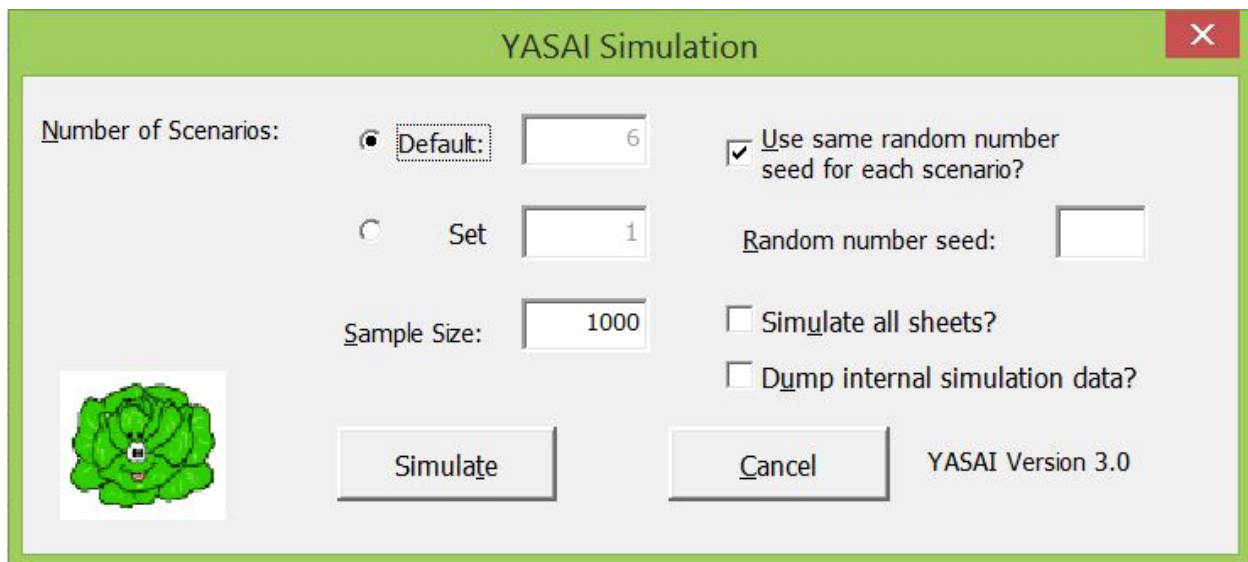


Figure 2.2 : YASAI Simulation input box window in Microsoft Excel[<http://www.yasai.rutgers.edu/yasai-guide-30.html>]

2.5 Random variates : Inverse Transform

A method used in this study for generating random number is Inverse Transform

Inverse Transform

From the inverse transform method for every U_i which are generated, corresponding X_i will be produced. Figure 13 illustrates the inverse transform technique utilizing the inverse of the CDF. it generates a number u_i between 0 and 1 find the corresponding x_i by using $F^{-1}(\cdot)$. For every values of u_i , the x_i will be properly “distributed”, one-to-one mapping between u_i and x_i [4].

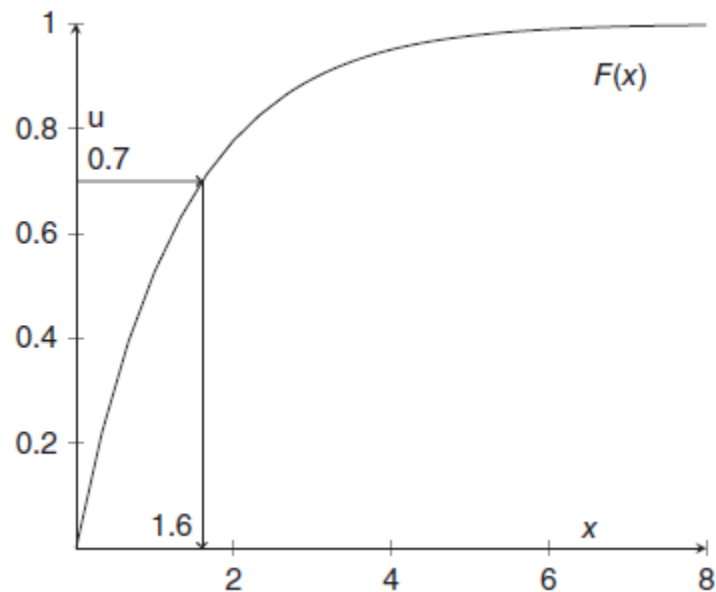


Figure 2.3 : Illustration of inverse transform method[Adopt from : [4]].

2.6 Central Limit Theorem

The theorem states that as the size of each sample gets large enough, the sampling distribution of the *mean* can be approximated by the normal distribution no matter what the distribution of the individual data might be[7].

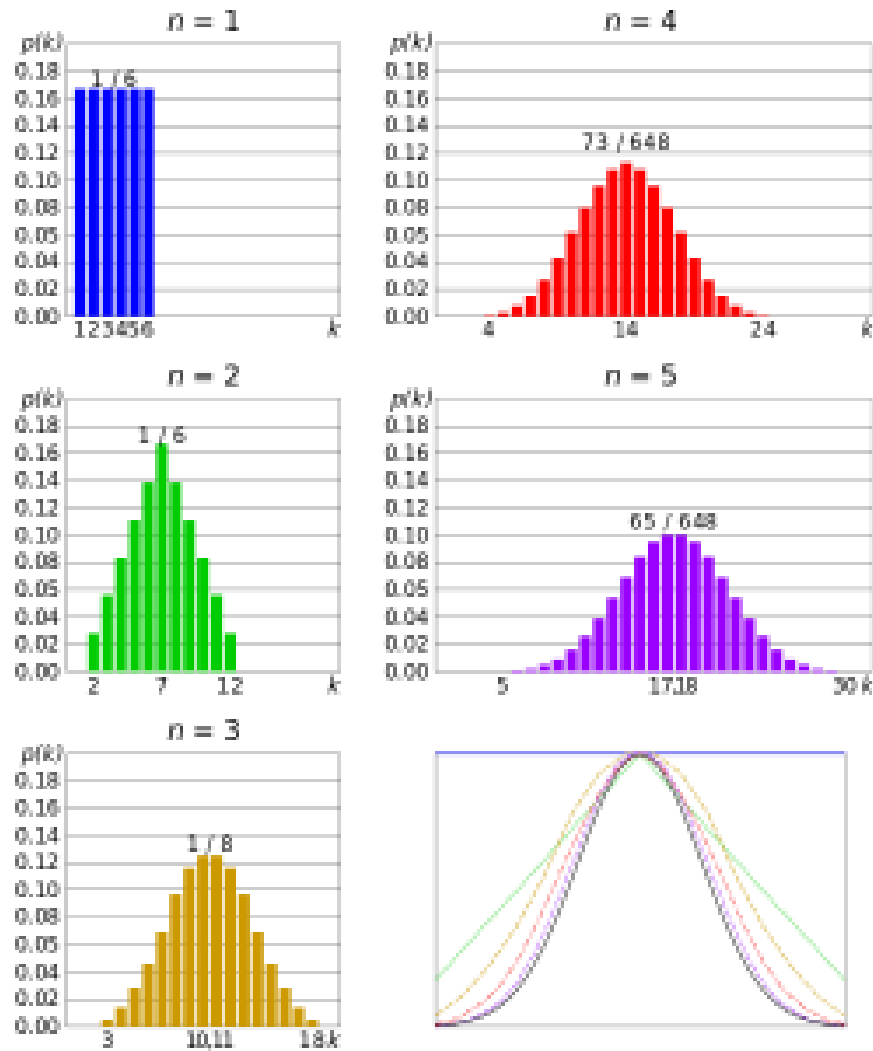


Figure 2.4 : Example of rolling the dice. [Adopt from : cmglee|Wikimedia Commons]

The distribution of the mean increasing, the more of the shape tend to be normal

2.7 t-Confidence Interval

Confidence intervals are used to get an estimation for a population parameter, usually the mean or population proportion[7].

If sample size is sufficiently large, an approximate 100 (1- α) percent confidence interval for μ is given by

$$\bar{x}(n) \pm Z_{1-\alpha/2} \sqrt{\frac{S^2(n)}{n}}$$

If sample size is too small, the alternative confidence interval, t distribution is applied

$$\bar{x}(n) \pm t_{n-1, 1-\alpha/2} \sqrt{\frac{S^2(n)}{n}}$$

Where $t_{n-1, 1-\alpha/2}$ is the upper 1- $\alpha/2$ critical point for the t distribution with n-1 degrees of freedom[3].

METHOD

The main method of this study is creating an algorithm and function of each step according to the flow chart below

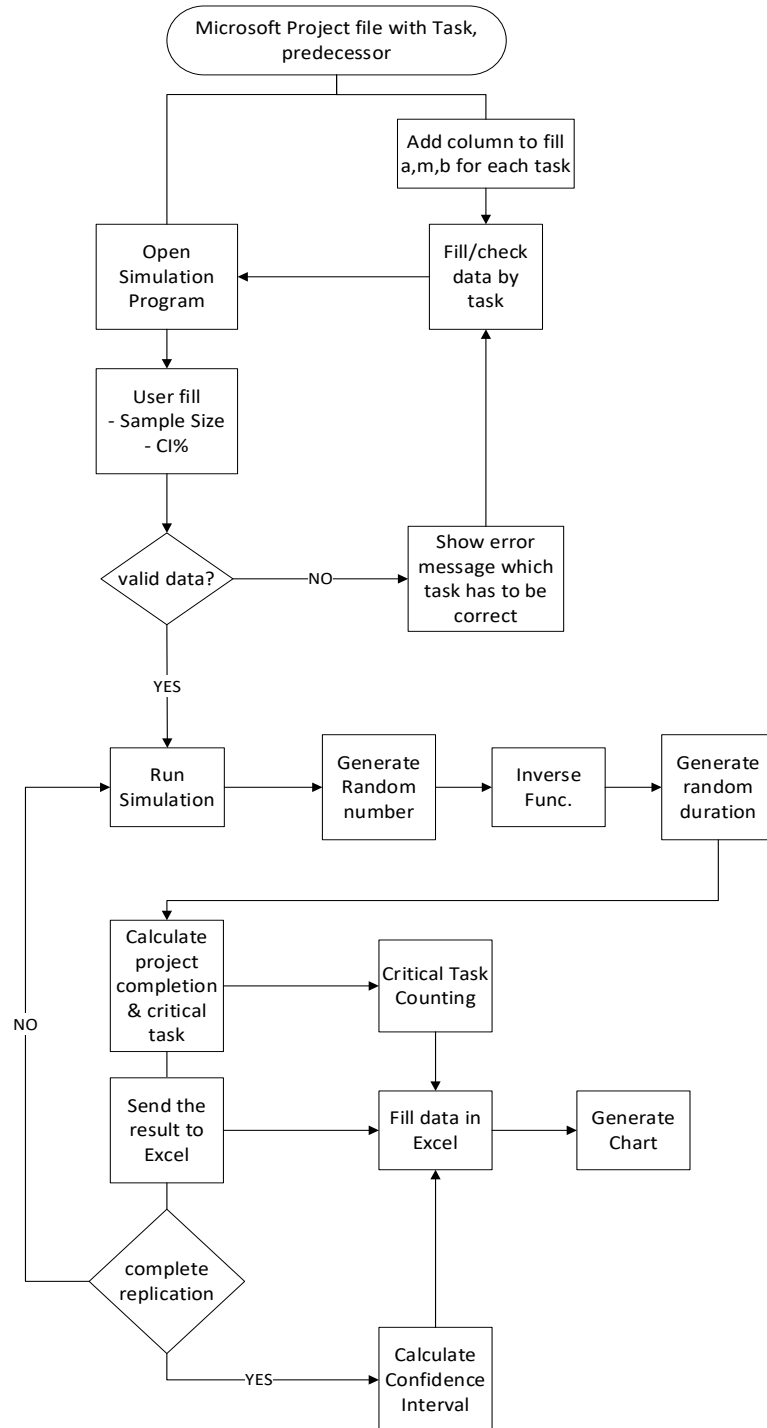


Figure 3.1 : Program execution flow chart

3.1 Study fundamental usage on Microsoft Project program.

Gantt Chart view is a default user interface window of Microsoft Project program, it contains 2 parts, Task Table and Gantt Chart. User frequently uses this window to input data that is a basic requirement from program; task name, task duration, predecessors and start-finish date of each task.

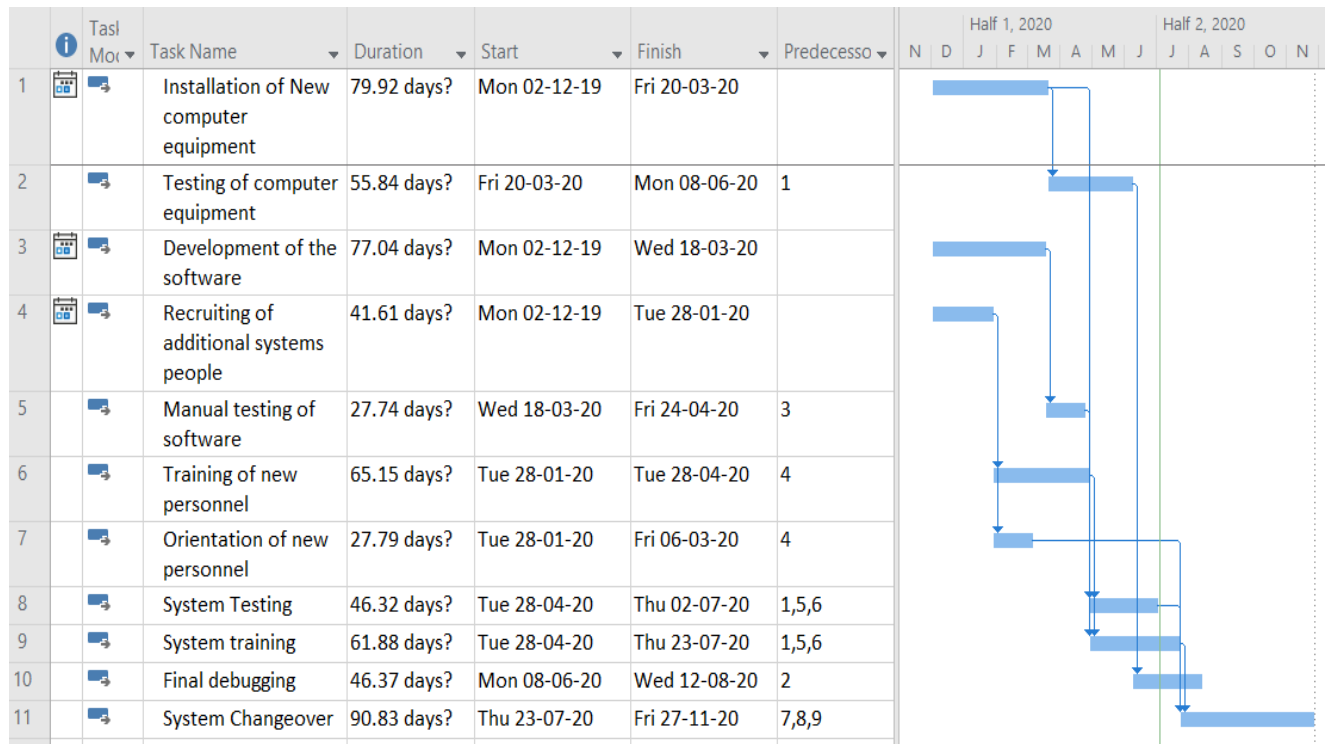


Figure 3.2 : Example of Task table and Gantt Chart view in Microsoft project

Therefore, the interface window for receiving new data to run simulation software will uses the default window, task table in Microsoft Project, as a filling input form, so the new data will be linked to existing task and convenience for user.

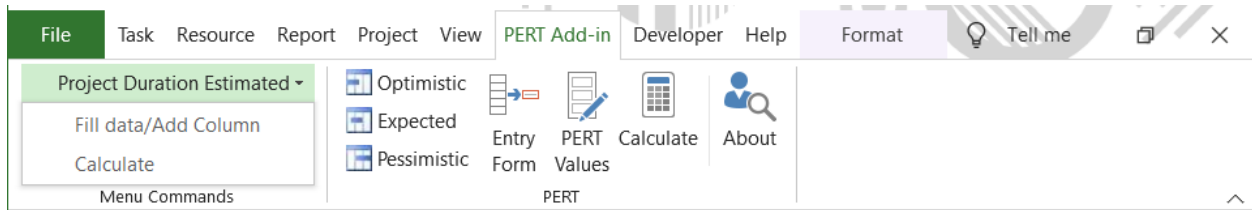


Figure 3.3 : Add-in menu for user to input data

Task No	Task Name	Duration	Start	Finish	Predecessor	Distributi	Min (a)	Avg (b)	Half 1, 2020					Half 2, 2020							
									J	F	M	A	M	J	J	A	S	O	N		
1	Installation of New computer equipment	79.92 days?	Mon 02-12-19	Fri 20-03-20		Normal	60 days	80 days													
2	Testing of computer equipment	55.84 days?	Fri 20-03-20	Mon 08-06-20	1	Constant	40 days	60 days													
3	Development of the software	77.04 days?	Mon 02-12-19	Wed 18-03-20		Normal	40 days	60 days													
4	Recruiting of additional systems people	41.61 days?	Mon 02-12-19	Tue 28-01-20		Uniform	30 days	40 days													
5	Manual testing of software	27.74 days?	Wed 18-03-20	Fri 24-04-20	3	Weibull	20 days	30 days													
6	Training of new personnel	65.15 days?	Tue 28-01-20	Tue 28-04-20	4	Weibull	40 days	70 days													
7	Orientation of new personnel	27.79 days?	Tue 28-01-20	Fri 06-03-20	4	Expo	20 days	30 days													
8	System Testing	46.32 days?	Tue 28-04-20	Thu 02-07-20	1,5,6	Expo	30 days	50 days													
9	System training	61.88 days?	Tue 28-04-20	Thu 23-07-20	1,5,6	Uniform	50 days	70 days													
10	Final debugging	46.37 days?	Mon 08-06-20	Wed 12-08-20	2	Beta	20 days	35 days													
11	System Changeover	90.83 days?	Thu 23-07-20	Fri 27-11-20	7,8,9	Triangle	60 days	100 days													

Figure 3.4 : Distribution menu in task table.

For the number of replication and confidence interval, user will input them in pop-up window after opening the simulation program since the normal task table window is not suitable for receiving these data.

Figure 3.5 : Pop-up window after opening the simulation program.

3.2 Create algorithm and function for running simulation following the step as follow

3.2.1 Checking the input data from user.

Create the program to simply checking the input data before running the simulation program to prevent incorrect result from calculation

1. Program to check the input data of min., average and max. duration from user.
2. Program to check the input distribution data, it must be selected for ever tasks.

3.2.2 Create random variable algorithm by using the algorithm from YASAI add-in program in Microsoft Excel. Test the algorithm by running and generating the result.

```
Function YRandom() As Double

Public s10, s11, s12, s20, s21, s22 As Double

Const a12 As Double = 1403580

Const a13n As Double = 810728

Const m1 As Double = 4294967087#

Const m2 As Double = 4294944443#

Const a21 As Double = 527612

Const a23n As Double = 1370589

Const norm As Double = 2.32830654929573E-10

If (s10 = 0 And s11 = 0 And s12 = 0) And (s20 = 0 And s21 = 0 And s22 = 0) Then

    s10 = 64785 , s11 = 3546, s12 = 123456, s20 = 658478, s21 = 73575

    s22 = 234567

End If

Dim k As Long

Dim p1, p2 As Double

p1 = a12 * s11 - a13n * s10

k = p1 / m1

p1 = p1 - (k * m1)

If (p1 < 0) Then

    p1 = p1 + m1

End If

s10 = s11

s11 = s12

s12 = p1

p2 = a21 * s22 - a23n * s20

k = p2 / m2

p2 = p2 - (k * m2)
```

```
Function YRandom() As Double (Cont'd)

If (p2 < 0) Then

    p2 = p2 + m2

End If

s20 = s21

s21 = s22

s22 = p2

If (p1 <= p2) Then

    YRandom = ((p1 - p2 + m1) * norm)

Else

    YRandom = ((p1 - p2) * norm)

End If

End Function
```

```
0.245901105726937
0.244786531179119
0.922201030845246
8.32975381812752E-02
0.216766594231001
0.848921611340646
0.199817191008939
0.464627080979393
0.370963450092915
0.707891983501971
0.280720322483645
0.372201815577685
0.214116244003219
0.961370542870154
0.35799680963702
0.126725778300074
0.560069347846887
0.809501391690293
7.97363632789729E-02
0.269750103146774
```

Figure 3.6 : Random variable generation algorithm and example of result that generated 20 times.

3.2.3 Study and create Inverse Function algorithm

Study inverse function of all probability distribution that was used in this study then create the inverse function program for each distribution, send the result back as the random value of duration of each activity.

For Beta distribution, its inverse function from Microsoft Excel will be used in this study.

Distribution	Program Output (Days)		
	Estimated Time	Mean (Result)	Variance (Result)
Normal	$\sqrt{-2 \times \ln R} \times \sin(2\pi R)$	$\frac{a + 4b + c}{6}$	$\left(\frac{c-a}{6}\right)^2$
Uniform	$a + (c-a)R$	$\frac{a+c}{2}$	$\frac{(c-a)^2}{12}$
Weibull	$\theta(-\ln R)^{\frac{1}{\beta}}$	$b - \theta \left(1 - \frac{1}{\beta}\right)^{\frac{1}{\beta}} + \theta \tau \left(1 + \frac{1}{\beta}\right)$	$\theta^2 \left\{ \tau \left(1 + \frac{2}{\beta}\right) - \left[\tau \left(1 + \frac{1}{\beta}\right) \right]^2 \right\}$
Beta	BETA.INV function(R,α,β,a,c) (Microsoft Excel function)	$\frac{a + 4b + c}{6}$	$\left(\frac{c-a}{6}\right)^2$
Triangle	if $R \leq \frac{b-a}{c-a}$; $\sqrt{\frac{2 \times R \times (b-a)}{c-a}} + a$ else ; $c - \sqrt{\frac{2 \times (1-R) \times (c-b)}{c-a}}$	$\frac{a+c+b}{3}$	$\frac{a^2 + b^2 + c^2 - ac - ab - cb}{18}$
Expo	$-b \times \ln R$	b	b^2
Constant	b	b	0

R = Random Number Generate (0,1)

$$\alpha = \left(\frac{2(c+4b-5a)}{3(c-a)} \right) \left(1 + 4 \left(\frac{(b-a)(c-b)}{(c-a)^2} \right) \right) \quad \text{for Beta distribution}$$

$$\beta = \left(\frac{2(5c-4b-a)}{3(c-a)} \right) \left(1 + 4 \left(\frac{(b-a)(c-b)}{(c-a)^2} \right) \right) \quad \text{for Beta distribution}$$

$$\beta = \frac{\ln(\ln(R(x_a)) / \ln(R(x_b)))}{\ln(a/c)} \quad \text{where } R(x_a) = 1 - 0.01 \quad R(x_b) = 1 - 0.99 \quad \text{Shape Parameter for Weibull distribution}$$

$$\theta = \frac{b}{(1 - 1/\beta)^{\frac{1}{\beta}}} \quad \text{Scale Parameter for Weibull distribution}$$

τ=Gamma Function (Weibull distribution)

Figure 3.7 : Inverse function formula of all distributions in this program

3.2.4 Sum all activities duration as a project duration

Create an array for collecting project duration result from 10 iteration of simulation. Every 10 values will be calculated to find the average of each replication.

3.2.5 Collect frequency of being critical of each task

Create an algorithm to count the frequency of being critical of each task for all replication. The counting result will be calculated the chance of being critical over all replications and then generated in Microsoft Excel.

$$\frac{\textit{frequency of critical task occurrence}}{\textit{Total replications}} \times 100\%$$

Microsoft Project automatically calculate critical task referring to each task duration and predecessor.

3.2.6 Create calculation function of confidence interval

Find an average of all project duration based on confidence interval provided by user, then generating the result through Microsoft Excel. The t confidence interval formula will be used.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Sample	Avg. Est. Duration		Percent % Est Completion	CI 95%			Critical Tsk.	% of occurrence			CI %	Lower Bound	Mean	Upper Bound	Stdev
1	101.3422917		0%	97.80			Meeting with board	100%			0.95	100.371226	101.599208	102.82719	2.62381226
2	101.8058333		10%	98.82			Interview/hire coaches	100%							
3	102.559375		20%	99.82			Reserve facilities	1%							
4	98.88729167		30%	100.14			Create schedule	73%							
5	108.7485417		35%	100.37	100.37	Lower Bound	Meeting with coaches	28%							
6	98.18729167		40%	100.84	100.84	Upper Bound	Register players	28%							
7	100.4875		50%	101.52	101.52	Mean	Order uniforms	73%							
8	103.1691667		51%	101.60	101.60	Mean	Collect fees	73%							
9	103.1685417		60%	101.76	101.76	Mean	Organize first Practice	100%							
10	101.7225		70%	102.57	102.57	Mean									
11	100.0120833		75%	102.83	102.83	Upper Bound									
12	101.7075		80%	103.13											
13	102.6075		90%	103.48											
14	101.07		100%	108.75											
15	97.80333333														
16	99.03145833														
17	100.1079167														
18	100.1529167														
19	106.2922917														
20	103.1208333														
21															

Figure 3.8 : An example of simulation result in Microsoft Excel Microsoft Project

3.2.7 Generate the result in chart format.

Using the chart function in Microsoft Excel to perform the chart of percentage(x-axis) versus estimated project duration(y-axis), including the area representation of confidence interval.

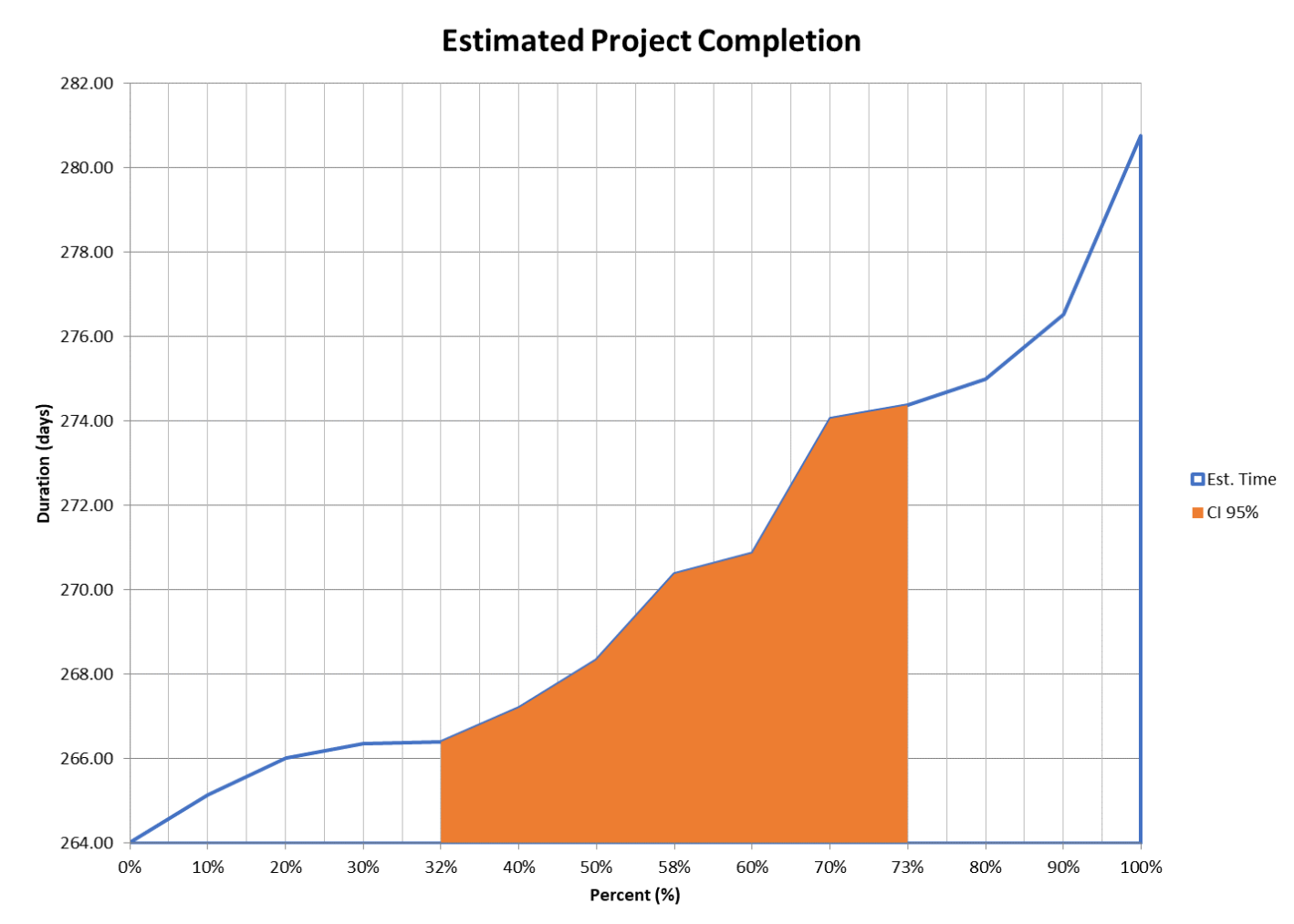


Figure 3.9 : An example of Chart representing the percentage, project duration and confidence interval

3.2.8 Create a chart to represent the sampling distribution of sample means if sample size get larger.

A whisker plot representing how the sampling distribution of sample means if the sample size got larger. So the simulation result is reliable if sampling distribution of larger sample dose not make any significantly different to default sample size(10 samples is default set in simulation program)

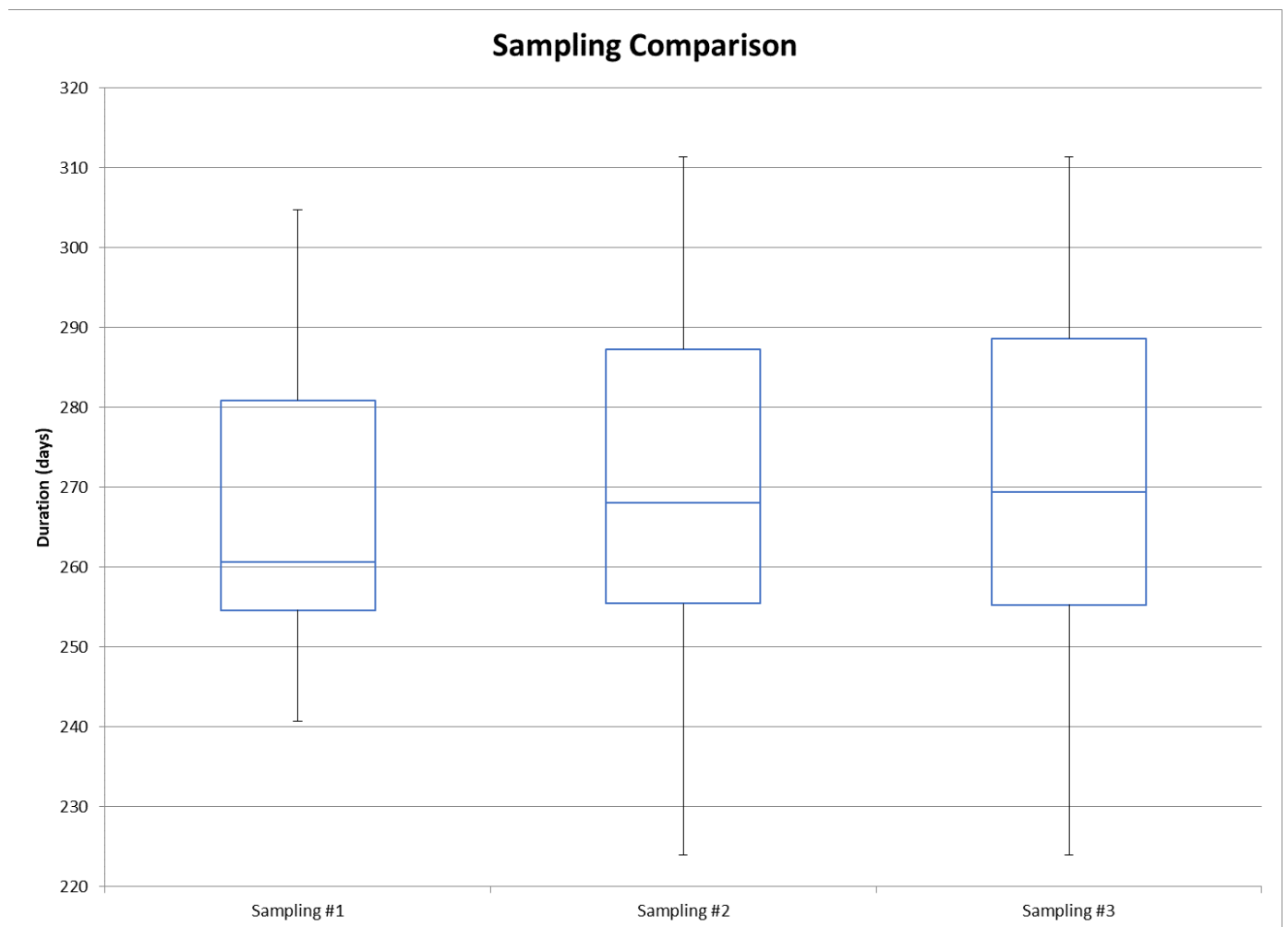


Figure 3.10 : An Example of Whisker plot of each sample size.

	Sampling #1	Sampling #2	Sampling #3
Count	10	51	101
Mean	267.736875	269.2439542	269.9993377
stdev	20.76067787	22.05846244	21.68509063
Min	240.3645833	223.6875	223.6875
Q1	254.2854167	255.159375	254.9760417
Median	260.2885417	267.75	269.0729167
Q3	280.5807292	286.9739583	288.3197917
Max	304.4145833	311.0291667	311.0291667
Bottom	254.2854167	255.159375	254.9760417
2Q Box	6.003125	12.590625	14.096875
3Q Box	20.2921875	19.22395833	19.246875
Whisker-	13.92083333	31.471875	31.28854167
Whisker+	23.83385417	24.05520833	22.709375

Table 1 : An example of statistical parameters of each sample size

3.2.9 Create histogram chart to represent the project duration data behavior.

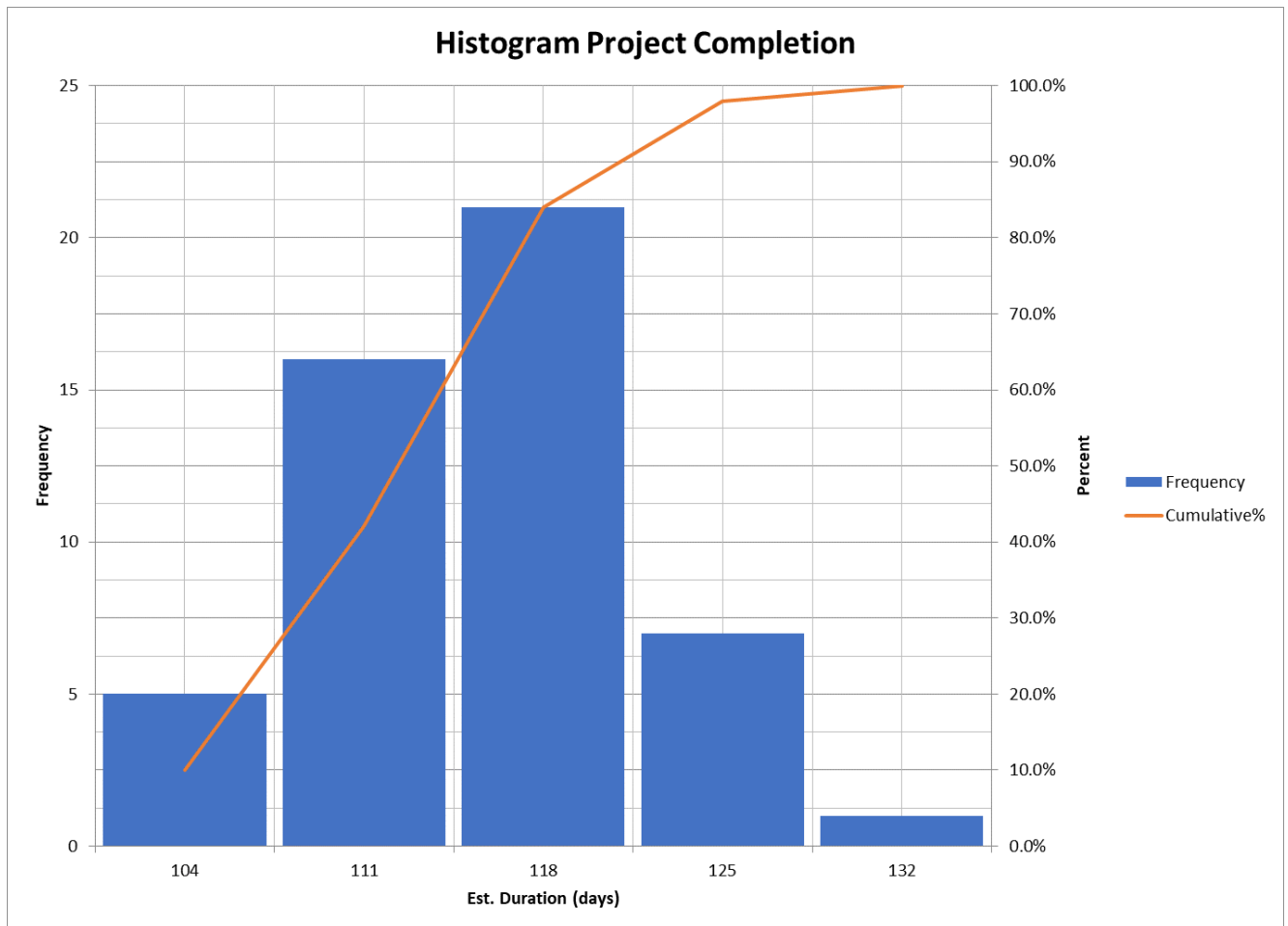


Figure 3.11 : An Example of Histogram plot of project duration data.

CASE STUDY

The case study shows the difference between CPM, PERT and Simulation program in generating the result and the flexibility of usage.

A project of single-storey office building is planned to be constructed with 15 activities. All activities, their immediate predecessors, and estimated times are as follows:

Activity	Description	Predecessors	Distribution	Duration (days)		
				Min (a)	Avg (b)	Max (c)
1	Excavation		Normal	2	4	6
2	Foundation	1	Beta	4	7	16
3	Rough Wall	2	Triangle	12	18	36
4	Company Signate	3	Normal	5	7	10
5	Roof	3	Weibull	8	11	20
6	Rough Exterior plumbing	5	Uniform	2	9	10
7	Exterior siding	5	Normal	8	9	20
8	Rough interior plumbing	6	Weibull	10	13	22
9	Exterior Painting	6,7	Beta	10	16	34
10	Wall boards	8	Triangle	6	15	18
11	Exterior Fixtures	9	Beta	3	18	26
12	Signate Installation	4,9	Uniform	2	4	7
13	Flooring	10	Triangle	8	15	25
14	Interior Painting	10,13	Normal	2	11	14
15	Interior Fixtures	11,14	Triangle	10	11	18

Table 2 : Activity time, distribution and their predecessors of the single-storey building project

[Adopt from : <https://www.quotemaster.org/critical+path>]

Based on Table 2, the project activities then arranged to a network regarding to its predecessor, as shown in Figure 4.1.

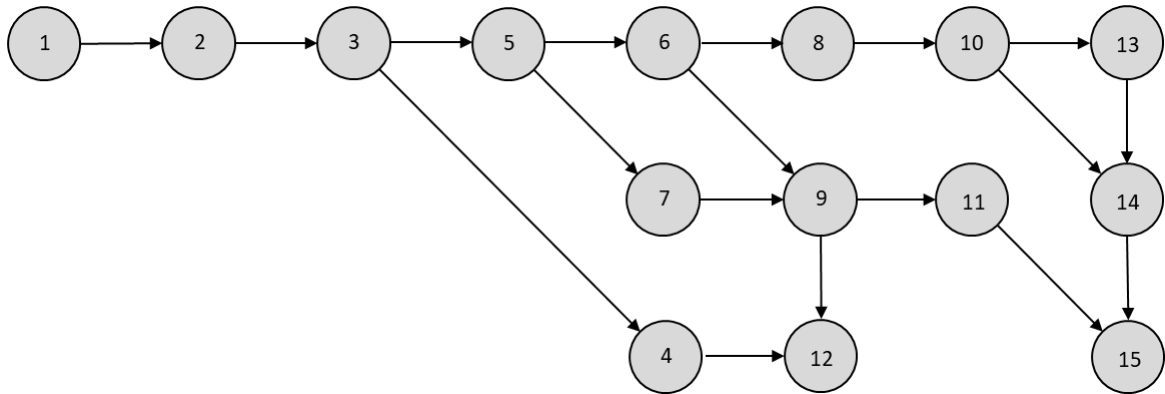


Figure 4.1 : Activity network of the case study project.

CPM, PERT and Project Simulation in Microsoft Project program will be performed to see the difference of project duration and critical task result.

4.1 CPM Solution

To find the critical task, the average duration of each activity in Table 2 will be used to calculate all of CPM parameters, Earliest and Latest of both starting and finishing time, and Slack time. The result is shown in Table 3.

Activity	Description	Predecessors	Duration (days)					Critical	
			Time	ES	EF	LS	LF		Slack
1	Excavation		4	0	4	0	4	0	Yes
2	Foundation	1	7	4	11	4	11	0	Yes
3	Rough Wall	2	18	11	29	11	29	0	Yes
4	Company Signate	3	7	29	36	96	103	67	No
5	Roof	3	11	29	40	29	40	0	Yes
6	Rough Exterior plumbing	5	9	40	49	40	49	0	Yes
7	Exterior siding	5	9	40	49	60	69	20	No
8	Rough interior plumbing	6	13	49	62	49	62	0	Yes
9	Exterior Painting	6,7	16	49	65	69	85	20	No
10	Wall boards	8	15	62	77	62	77	0	Yes
11	Exterior Fixtures	9	18	65	83	85	103	20	No
12	Signate Installation	4,9	4	65	69	110	114	45	No
13	Flooring	10	15	77	92	77	92	0	Yes
14	Interior Painting	10,13	11	92	103	92	103	0	Yes
15	Interior Fixtures	11,14	11	103	114	103	114	0	Yes

Table 3 : Start and finish times, and slack by using CPM method

From Table 3, the total project duration is 114 days corresponding to the critical path which is shown in Figure 4.2.

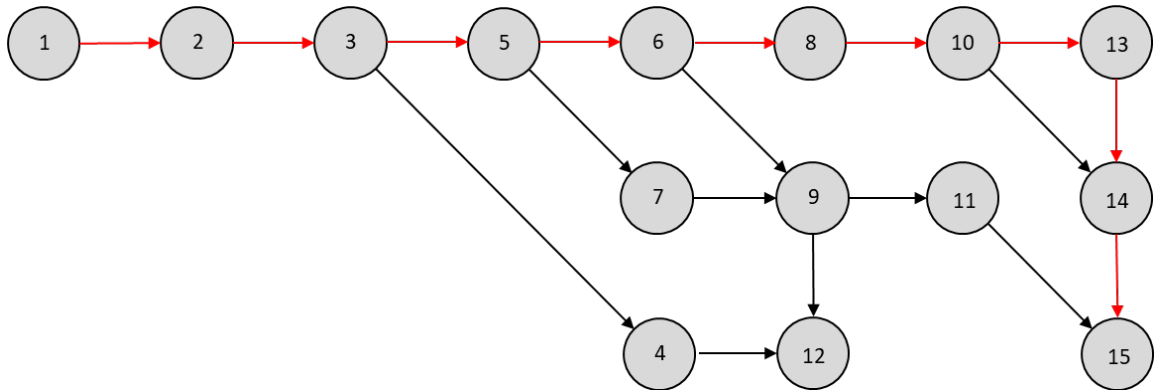


Figure 4.2 : Critical path by CPM method

4.2 PERT Solution

The duration of each activity will be considered as expected duration time which follow Beta distribution. According to 3 points estimation: min, average and max in Table 2, the result of PERT calculation and critical path are shown in Table 4 and Figure 4.3 respectively.

Activity	Description	Predecessors	Duration (days)						Critical	Expected Duration
			Time	ES	EF	LS	LF	Slack		Mean
1	Excavation		4.00	0	4	0	4	0	Yes	4.00
2	Foundation	1	8.00	4	12	4	12	0	Yes	8.00
3	Rough Wall	2	20.00	12	32	12	32	0	Yes	20.00
4	Company Signate	3	7.17	32	39	98	106	66	No	7.17
5	Roof	3	12.00	32	44	32	44	0	Yes	12.00
6	Rough Exterior plumbing	5	8.00	44	52	44	52	0	Yes	8.00
7	Exterior siding	5	10.67	44	55	60	71	16	No	10.67
8	Rough interior plumbing	6	14.00	52	66	52	66	0	Yes	14.00
9	Exterior Painting	6,7	18.00	55	73	71	89	16	No	18.00
10	Wall boards	8	14.00	66	80	66	80	0	Yes	14.00
11	Exterior Fixtures	9	16.83	73	90	89	106	16	No	16.83
12	Signate Installation	4,9	4.17	73	77	113	118	41	No	4.17
13	Flooring	10	15.50	80	96	80	96	0	Yes	15.50
14	Interior Painting	10,13	10.00	96	106	96	106	0	Yes	10.00
15	Interior Fixtures	11,14	12.00	106	118	106	118	0	Yes	12.00

Table 4 : Start and finish times, and slack by using PERT method

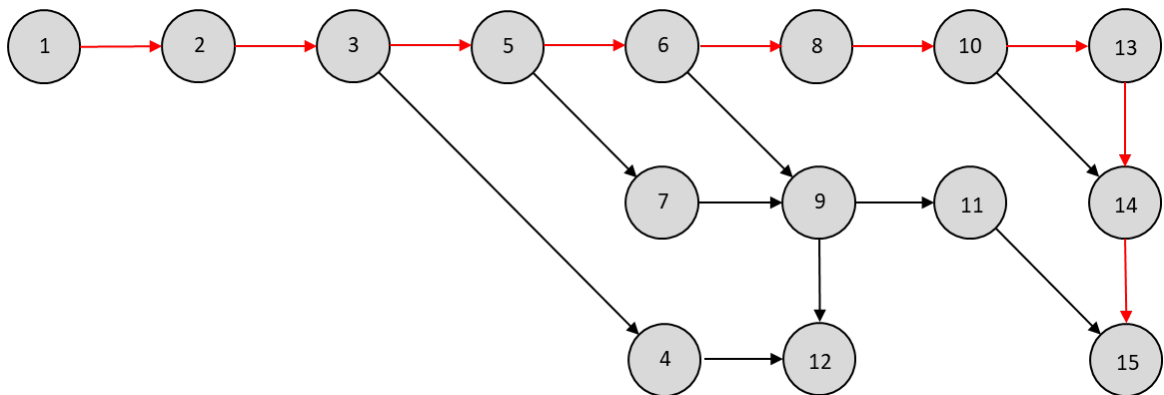


Figure 4.3 : Critical path by PERT method

As the result, the critical path from PERT and CPM calculation are the same as shown in Figure 4.2. However, the total project duration of 118 days of PERT is 4 days longer than CPM due to the uncertainty of each activity.

4.3 Simulation solution

The calculation is performed by following each distribution as shown in Table 2.

With 20 sample size and 95 percent confidence interval:

1. Estimated project duration with 95 percent confidence interval is 118.08 ± 2.81 days
2. The possibility of being critical of each task presented in Table 5

Critical Tsk.	% of occurrence
Excavation	100%
Foundation	100%
Rough Wall	100%
Company Signate	0%
Roof	100%
Rough Exterior plumbing	73%
Exterior siding	27%
Rough interior plumbing	73%
Exterior Painting	27%
Wall boards	73%
Exterior Fixtures	27%
Signate installation	0%
Flooring	73%
Interior Painting	73%
Interior Fixtures	100%

Table 5 : The possibility of being critical of each task from simulation method

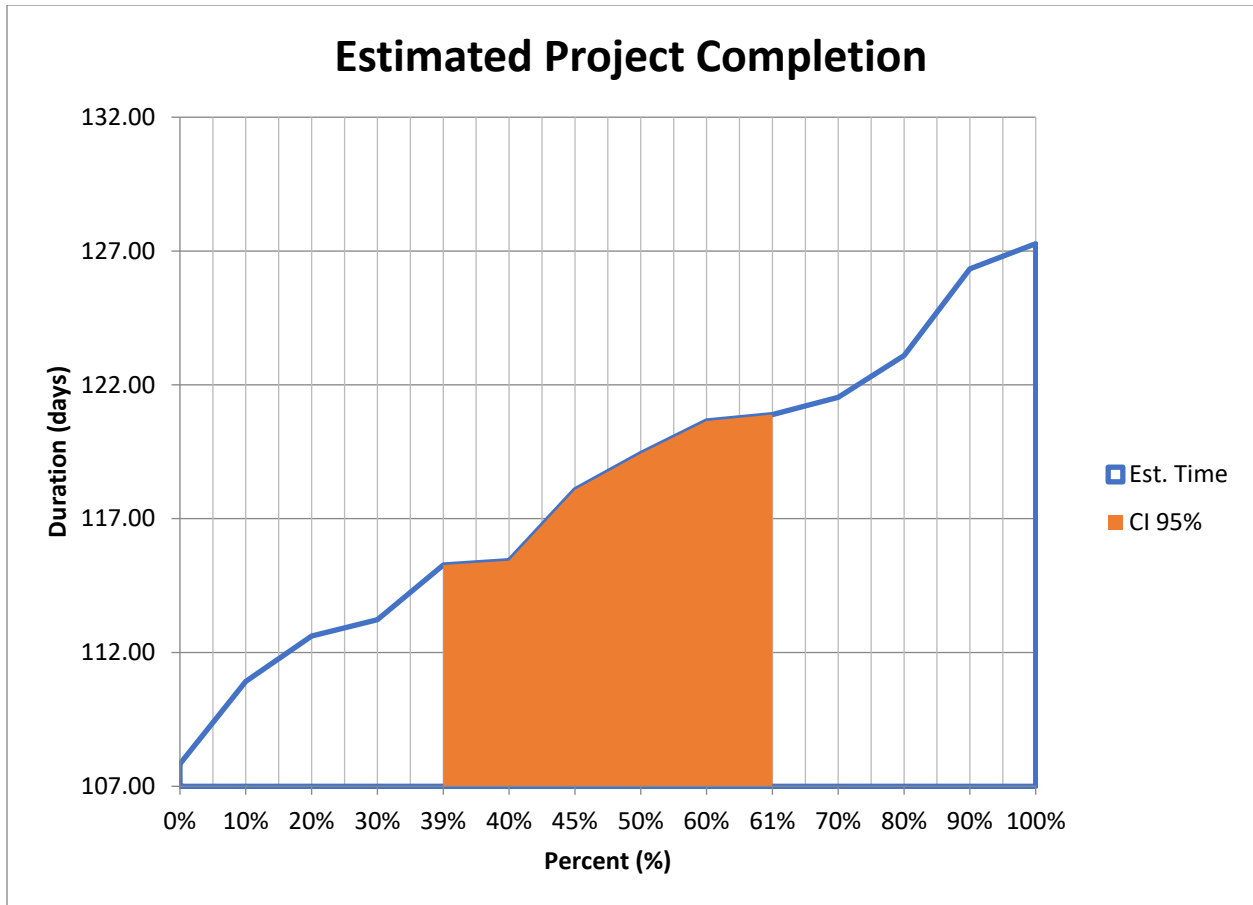


Figure 4.4 : Estimated project duration chart and 95% of confidence interval

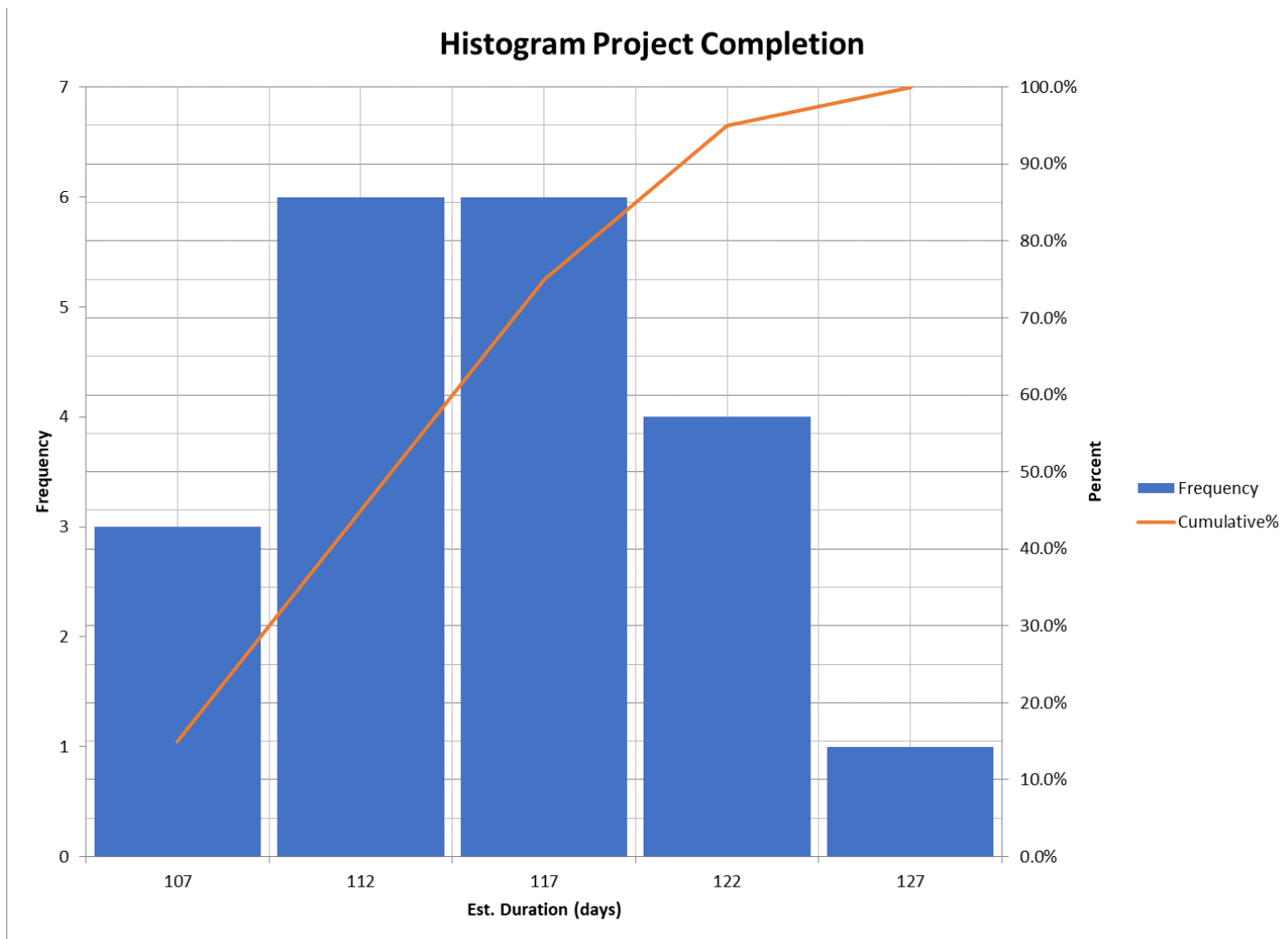


Figure 4.5 : Histogram chart and cumulative frequency of all sample data

4.4 Conclusion

CPM and PERT program will not generate the result from variety of probability distribution(no distribution option) and only one result of project completion time is generated which is 114 and 118 days while simulation program run and calculate the project duration from probability distribution of each task and also generate the result by interval which is 118.08 ± 2.81 days. Therefore the project duration is more precisely predicted within 95% confidence interval.

For critical task, CPM and PERT technique give us only one critical path while simulation technique gives us possibility of being critical of each task. For example, according to table 5, Exterior siding(7), Exterior Painting(9) and Exterior Fixture(11) task are about 27% chance of being critical that it may cause the new critical path as shown in Figure 4.5. Therefore, there are approximately 27% of uncertainty which causing the project delay.

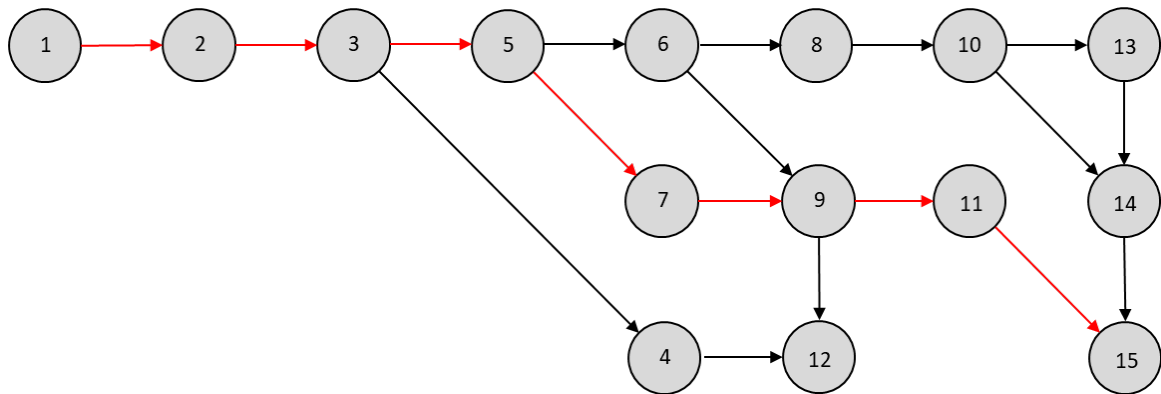


Figure 4.6 : Critical path for 27% chance of being critical

Thus, the project simulation technique give us project duration regarding to all possible critical tasks. This will help project manager to carefully manage the project by not focusing just only tasks that are truly the most critical.

CONCLUSIONS

The project simulation tool gives us more details about critical path occurrence and project duration from variety distributions. This tool improves the solution that CPM and PERT cannot such as

1. Some activities may have a little chance of being critical and may delay a project. The CPM and PERT tools may not reveal these activities as a major critical
2. PERT tool generates the project duration from average duration of each activity. Its result doesn't cover an overview of possible uncertainty from activities variation that affect the project completion.
3. Normal and Beta distribution are mostly used in PERT; so some activity may consider be different from its actual distribution and may generate misleading result.

As result, by using project simulation, project managers can foresee another possibility of being critical, not only some activities which are fall on major critical path. Consequently, they will prepare the action plan and manage the project properly.

The simulation program will be more effective if it able to run with widely used program, so users are able to conduct the simulation together with current task management program without switching programs or transferring the file to another program. Microsoft Project is the proper choice for this study. The current version of this program does not include simulation program as a default function in standard package, fortunately, user can create custom program by using VBA (Visual Basic Application) and this can be also ran with another programs such as Microsoft Excel.

The obvious difference between simulation and PERT is the result from variety distributions selection in simulation program. User has to identify distribution of each activity means while the package PERT in Microsoft Program does not require this information. User may use both tools, simulation and PERT, to perform the project scheduling and making a decision based on both outcomes.

FURTHER STUDY

This simulation program may well suited especially for starting project planning and all tasks are independent (no correlation between task) while many on progress projects; some activities might have variation factors causing project delay and need to be rescheduled. This software now cannot support this kind scenario. Hence, the recommended further study is perhaps about improving the program capable for calculating the remaining time in each activity and project duration while running the project. The result may answer the question “How long the project will be completed if the actual progress is updated?”

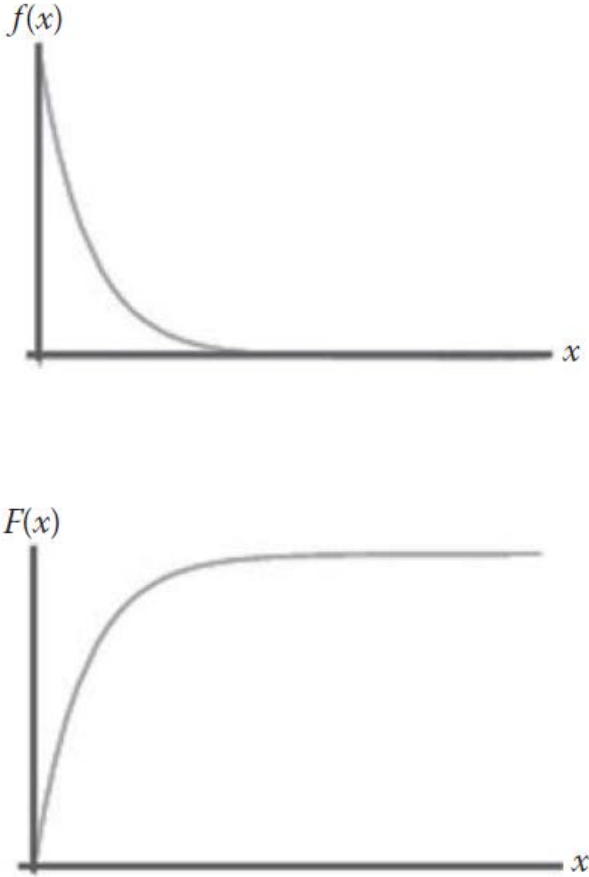
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APPENDIX

Continuous Probability Distribution

Exponential	
Distribution	This distribution is often used to model intervene times in random arrival and breakdown processes, but it is generally inappropriate for modeling process delay times.
PDF	$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$
CDF	$F(x) = \int_{-\infty}^x f(x)dx = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$
Mean	$\frac{1}{\lambda}$
Variance	$\frac{1}{\lambda^2}$
Inverse function	$-b \times \ln R$
Appearance	

*R = Random Number Generate (0,1)

Beta

Distribution

Due to its ability to take on a wide variety of shapes, this distribution is often used as a rough model in the absence of data. Because the range of the beta distribution is from 0 to 1, the sample X can be transformed to the scaled beta sample Y with the range from a to b by using the equation $Y = a + (b-a)X$. The beta is often used to represent random proportions such as the proportion of defective items in a lot.

PDF

$$f(x) = \begin{cases} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}, \text{ where}$$

B is the complete beta function and is given by

$$B(\alpha,\beta) = \int_0^1 t^{\beta-1}(1-t)^{\alpha-1} dt$$

CDF

$$F(X; \alpha, \beta) = \frac{B(X; \alpha, \beta)}{B(\alpha, \beta)} = I_x(\alpha, \beta), \text{ where}$$

$B(X; \alpha, \beta)$ is the complete beta function and
 $I_x(\alpha, \beta)$ is the regularized incomplete beta function

Mean

$$\frac{\alpha}{\alpha + \beta}$$

Variance

$$\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

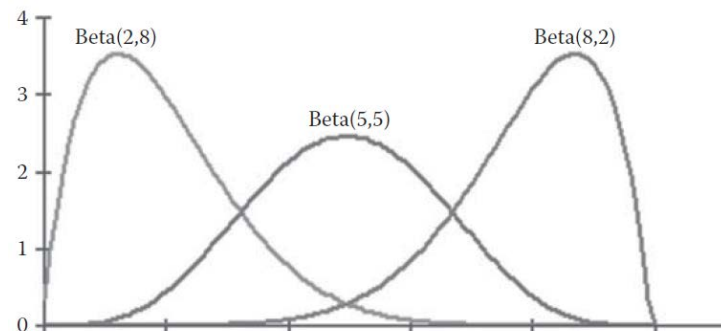
Inverse function

BETA.INV function(R, α, β, a, c)
 (Microsoft Excel function)

$$\alpha = \left(\frac{2(c + 4b - 5a)}{3(c - a)} \right) \left(1 + 4 \left(\frac{(b - a)(c - b)}{(c - a)^2} \right) \right)$$

$$\beta = \left(\frac{2(5c - 4b - a)}{3(c - a)} \right) \left(1 + 4 \left(\frac{(b - a)(c - b)}{(c - a)^2} \right) \right)$$

Appearance



*R = Random Number Generate (0,1)

Normal

Distribution This distribution is used in situations in which the central limit theorem applies, that is, quantities that are the sums of other quantities. It is also used empirically for many processes that appear to have a symmetric distribution.

PDF

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ and σ are the mean and standard deviation, respectively. For $\mu=0$ and variance σ^2 , the distribution is called standard normal distribution.

CDF

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

(For standard normal distribution)

Mean

μ

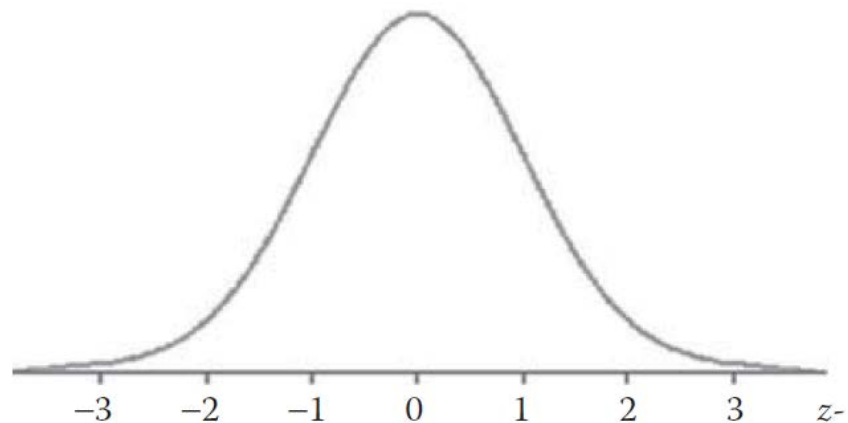
Variance

σ^2

Inverse function

$$\sqrt{-2 \times \ln R} \times \sin(2\pi R)$$

Appearance



*R = Random Number Generate (0,1)

Triangular

Distribution

This distribution is commonly used in situations in which the exact form of the distribution is not known, but estimates for the minimum, maximum, and most likely values are available. This distribution is easy to use and explain than other distributions that may be used in this situation.

PDF

$$f(x) = \begin{cases} 0, & x < a \\ \frac{2(x-a)}{(b-a)(c-a)}, & a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)}, & c < x \leq b \\ 0, & b < x \end{cases},$$

where a , b , and c are constants; $a < b$ and $c \leq b$

CDF

$$F(x) = \begin{cases} 0, & x < a \\ \frac{(x-a)^2}{(b-a)(c-a)}, & a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)}, & c < x \leq b \\ 1, & b < x \end{cases}$$

Mean

$$\frac{a+b+c}{3}$$

Variance

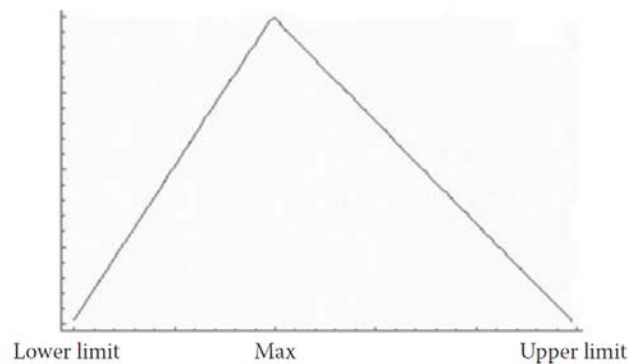
$$\frac{a^2 + b^2 + c^2 + ab + bc + ca}{18}$$

Inverse function

$$\text{if } R \leq \frac{b-a}{c-a}; \sqrt{\frac{2 \times R \times (b-a)}{\frac{2}{c-a}}} + a$$

$$\text{else ; } c - \sqrt{\frac{2 \times (1-R) \times (c-b)}{\frac{2}{c-a}}}$$

Appearance



*R = Random Number Generate (0,1)

Weibull

Distribution

The distribution is widely used in reliability models to represent the lifetime of a device. If a system consists of a large number of part that fail independently, and if the system fails when any single part fails, then the time between successive failures can be approximated by a Weibull distribution. This distribution is also used to represent nonnegative task times that are skewed to the left.

PDF

$$f(x) = \begin{cases} \alpha \beta^{-\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

CDF

$$F(X) = \begin{cases} 1 - e^{-(x/\beta)^\alpha}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Mean

$$\frac{\beta}{\alpha} \Gamma\left(\frac{1}{\alpha}\right)$$

Variance

$$\frac{\beta^2}{\alpha} \left\{ 2\Gamma\left(\frac{2}{\alpha}\right) - \frac{1}{\alpha} \left[\Gamma\left(\frac{1}{\alpha}\right) \right]^2 \right\}$$

Inverse function

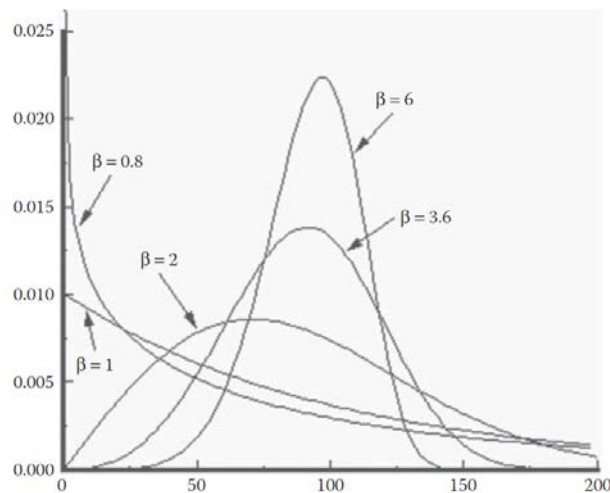
$$\theta(-\ln R)^{\frac{1}{\beta}}$$

$$\beta = \frac{\ln(\ln(R(x_a)) / \ln(R(x_b)))}{\ln(a/c)} \quad \text{where } \begin{matrix} R(x_a) = 1 - 0.01 \\ R(x_b) = 1 - 0.99 \end{matrix}$$

$$\theta = \frac{b}{(1 - 1/\beta)^{\frac{1}{\beta}}} \quad \text{Scale Parameter for Weibull distribution}$$

*R = Random Number Generate (0,1)

Appearance



Uniform U(a,b)

Distribution Used as a "first" model for a quantity that is felt to be randomly varying between a and b but about which little else is known. The U(0,1) distribution is essential in generating random values from all other distributions.

PDF

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

CDF

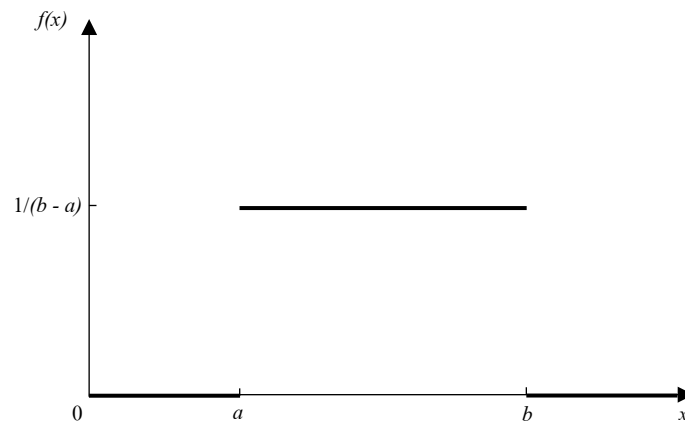
$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b < x \end{cases}$$

Mean $\frac{a+b}{2}$

Variance $\frac{(b-a)^2}{12}$

Inverse function $a + (c-a)R$

Appearance



U(a,b) density function.

*R = Random Number Generate (0,1)